Straight lines

1. Find the equation of the straight line which passes through the point $(1, -2)$ and cuts off equal intercepts from axes.
2. Find the equation of the line passing through the point $(5, 2)$ and perpendicular to the line joining the points $(2, 3)$ and $(3, -1)$.
3. Find the angle between the lines $y = (2 - \sqrt{3})(x + 5)$ and $y = (2 + \sqrt{3})(x - 7)$.
4. Find the equation of the lines which passes through the point $(3, 4)$ and cuts off intercepts from the coordinate axes such that their sum is 14.
5. Find the points on the line $x + y = 4$ which lie at a unit distance from the line $4x + 3y = 10$.
6. Show that the tangent of an angle between the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{a} - \frac{y}{b} = 1$ is $\frac{2ab}{a^2 - b^2}$.
7. Find the equation of lines passing through $(1, 2)$ and making angle $30°$ with y-axis.
8. Find the equation of the line passing through the point of intersection of $2x + y = 5$ and $x + 3y + 8 = 0$ and parallel to the line $3x + 4y = 7$.
9. For what values of a and b the intercepts cut off on the coordinate axes by the line $ax + by + 8 = 0$ are equal in length but opposite in signs to those cut off by the line $2x - 3y + 6 = 0$ on the axes.
10. If the intercept of a line between the coordinate axes is divided by the point $(-5, 4)$ in the ratio $1 : 2$, then find the equation of the line.
11. Find the equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of $120°$ with the positive direction of x-axis.

    [Hint: Use normal form, here $\omega = 30°$.]
12. Find the equation of one of the sides of an isosceles right angled triangle whose hypotenuse is given by $3x + 4y = 4$ and the opposite vertex of the hypotenuse is $(2, 2)$.

13. If the equation of the base of an equilateral triangle is $x + y = 2$ and the vertex is $(2, -1)$, then find the length of the side of the triangle.

[**Hint**: Find length of perpendicular (p) from $(2, -1)$ to the line and use $p = l \sin 60°$, where l is the length of side of the triangle].

14. A variable line passes through a fixed point P. The algebraic sum of the perpendiculars drawn from the points $(2, 0)$, $(0, 2)$ and $(1, 1)$ on the line is zero. Find the coordinates of the point P.

[**Hint:** Let the slope of the line be m. Then the equation of the line passing through the fixed point $P(x_1, y_1)$ is $y - y_1 = m(x - x_1)$. Taking the algebraic sum of perpendicular distances equal to zero, we get $y - 1 = m(x - 1)$. Thus (x_1, y_1) is $(1, 1)$.]

15. In what direction should a line be drawn through the point $(1, 2)$ so that its point of intersection with the line $x + y = 4$ is at a distance $\frac{\sqrt{6}}{3}$ from the given point.

16. A straight line moves so that the sum of the reciprocals of its intercepts made on axes is constant. Show that the line passes through a fixed point.

[**Hint:** $\frac{x}{a} + \frac{y}{b} = 1$ where $\frac{1}{a} + \frac{1}{b} =$ constant $= \frac{1}{k}$ (say). This implies that $\frac{k}{a} + \frac{k}{b} = 1 \Rightarrow$ line passes through the fixed point (k, k).]

17. Find the equation of the line which passes through the point $(-4, 3)$ and the portion of the line intercepted between the axes is divided internally in the ratio $5 : 3$ by this point.

18. Find the equations of the lines through the point of intersection of the lines $x - y + 1 = 0$ and $2x - 3y + 5 = 0$ and whose distance from the point $(3, 2)$ is $\frac{7}{5}$.

19. If the sum of the distances of a moving point in a plane from the axes is 1, then find the locus of the point. [**Hint**: Given that $|x| + |y| = 1$, which gives four sides of a square.]

20. P_1, P_2 are points on either of the two lines $y - \sqrt{3}\,|x| = 2$ at a distance of 5 units from their point of intersection. Find the coordinates of the foot of perpendiculars drawn from P_1, P_2 on the bisector of the angle between the given lines.

[**Hint**: Lines are $y = \sqrt{3}x + 2$ and $y = -\sqrt{3}\,x + 2$ according as $x \geq 0$ or $x < 0$. y-axis is the bisector of the angles between the lines. P_1, P_2 are the points on these lines at a distance of 5 units from the point of intersection of these lines which have a point on y-axis as common foot of perpendiculars from these points. The y-coordinate of the foot of the perpendicular is given by $2 + 5\cos 30°$.]

21. If p is the length of perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$ and a^2, p^2, b^2 are in A.P, then show that $a^4 + b^4 = 0$.